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Applications of Fuzzy Set

and Probability Theory

to Naval Ocean Surveillance

Dr. I. R. Goodman, Code 7232

Development Support Branch

Surveillance Systems Department

Naval Ocean Systems Center, Bayside

San Diego, California 92152

June, 1981

47th MORS

ABSTRACT OF PAPER

The problem of multi-target, multi-sensor Naval Ocean-Surveillance Correlation (or data association) is treated from a combined probabilistic and fuzzy set viewpoint. The basic figure of merit used in determining the accuracy of a correlator-tracker is the posterior distribution of data partitionings. This is shown to decompose into a product of conditional factors, one representing geolocation goodness-of-fit, another false alarms, and another, target attribute information which is often given originally in linguistic form. In modeling the last factor here, an expert-queried rule-based system is first developed. This is then translated into a conjunction of compound statements consisting of probabilistic or fuzzy set components. These are all converted first to a fuzzy set structure, with fuzzy logic employed, yielding a conclusion set. Finally, using the author's previous discovery linking fuzzy and random sets-through a simple canonical mapping which induces a homomorphism between most fuzzy and random set operations - the results are translated back into a probabilistic framework, giving a probabilistic description of the attribute factor.

Outline of Proposed Paper

for Presentation at the 47th MORS

Ft. McNair, Wash., D.C.

July 7-9, 1981

This paper treats the problem of modeling multi-target, multi-sensor data association or correlation. The overall problem of multi-target tracking and correlation in Naval Ocean Surveillance continues to be a formidable challenge, despite many intensive efforts by both private industry and Government. (See, e.g., proceedings from the past several years' sessions of the IEEE Conference on Decision and Control, the MIT/ONR Symposia on Distributed Information and Decision Systems Motivated by Command-Control-Communication Problems, and the Naval Research Laboratory's Naval Ocean-Surveillance Correlation Handbook and Library.)

An important step in treating the correlation problem has been the recognition that it can be considered formally a data partitioning problem with unknown parameter Q being the true partitioning or association of objects into track sets or false alarms. (See the works of Sittler, Tse, Bar-Shalom, Reid, Morefield, Goodman, Stein, Blackman, and others in various IEEE publications.) In particular, the posterior distribution $pr(Q|Y)$ conditioned on data set Y - or some reasonable approximation of this quantity - is the key measure in deriving and measuring the performance of correlator-tracker schemes. $pr(Q|Y)$ is shown to decompose into a product of goodness-of-fit factors, which can vary somewhat in structure, depending if forms are sought which are either recursive or nonrecursive with respect to the use of incoming new data relative to previously treated data. In any case, the goodness-of-fit factors are essentially: (1) geolocational - related to a weighted sum of squared Kalman filter innovations or residuals; (2) false alarms - modeled often by a Poisson dispersal population, a uniformly distribution, or a truncated Gaussian distribution; and (3) attributes. Because of the often non-numerical or vague linguistic descriptions involved in (3), modeling of these components can become difficult. For example, intelligence information may indicate a target is in the approximate area, or a sensor reading of a signal is deemed to be target-like, and following an irregular zig-zag pattern with large deviations. Or, perhaps an operator may state in his report that "we believe the target tracker is now most likely in bearing line range a, but is also possibly in the bearing line region b." The technique introduced here easily incorporates this vague-linguistic information directly into $pr(Q|Y)$. Furthermore, the procedure can be extended to be, in effect, an alternative approach to the modeling of correlation.

First, a rule-based system is established. This consists of a collection of modus ponens (or implicational forms 'if ( ), then ( )') rules which surveillance operators usually use in concluding that new data associates, or not, with previously established tracks, or with each other. In practice, the rules are obtained only after querying a sufficient number of operators, but weeding-out as many inconsistencies as possible. A typical rule for comparing two objects - say a new data point and a previously established (and updated) track set - will have the following form: "If attribute values $A_1,..,A_n$ are obtained for object one, and values $B_1,..,B_m$, for object two, then it is possible that objects one and two associate with the same platform." Another rule might be more negative: "If attribute A' (say bearing measurement) holds for object one

and attribute value B' (say two dimensional position fix with associated uncertainty region) holds for object two (previously established track) then one and two correlate with moderate probability, only if A' intersects B' and the measured range distance to the sensor source is still effective."

In addition to the rules, the system consists (depending on sampling time and local sensor systems employed) of statements describing which attributes are present and what their perceived values are. The component parts of both the modus ponens rules and the attribute data may be describable most naturally in either mathematical-probabilistic form or in linguistic (possible vague) form, the latter employing typical hedges or emphasizers such as 'most', 'usually', 'often', probably', 'possibly', 'with high confidence', 'most likely', etc. In the latter case, it is most convenient to use the techniques of fuzzy set theory in the modeling.

In a series of previous papers (see especially NRL report 8415 and forthcoming book "Recent Developments in Fuzzy Set Theory and Possibility Theory", Peugamon Press), the author has shown that an explicit connection exists between the fuzzy set approach and that of the more well-known probabilistic-statistical approach to the modeling of uncertainty. A relatively simple mapping was derived (since then, an additional mapping has been discovered) which connects fuzzy sets with random sets. As a consequence, every fuzzy set is shown to be equivalent - through its membership function - to an entire class of corresponding random sets - through their common one-point coverage function. In addition, the mapping is shown to induce a homomorphism between all of the major operations of fuzzy set theory and that of ordinary random (and non-random) sets. In essence, this implies that modeling, where convenient, can be carried out in a fuzzy setting, using all of the many established fuzzy techniques available, and then reinterpreted in a rigorous manner in the usual probabilistic setting.

In particular, applying the above results to logical reasoning yields the following: The rule-based system modeling the attribute aspect of the correlation problem is a conjunction of modus ponens statements and other statements compounded from atoms of fuzzy sets or probabilistic statements (in the form of random sets). Consequently, the entire system may be modeled in a fuzzy set framework determining a single (but compound) fuzzy set statement $\phi_B(V)\geq\tau$, where fuzzy set membership function ϕ_B is computable from standard fuzzy set manipulation techniques, with max-min operations predominating. V is an unknown parameter vector representing in one component Q, and in other components, the measurement dimensions yielding the various attribute values. τ is the confidence set level. Carrying out a projection operation, in turn, yields $\phi_C(Q)\geq\tau$, where fuzzy set C is computed from B. The last expression is a fuzzy confidence set for the possible values of Q. This confidence set is seen to be a <u>uniformly most accurate</u> description of Q. That is, it can be shown (related to the previous mentioned mapping connecting fuzzy and random sets) that this description of possible Q's makes maximal use of the system's information. Finally, making direct use of the canonical mapping, a probabilistic description is obtained for Q: $Pr(Q\in S_U(C)|Y)\geq\tau$, where $S_U(C)$ is a computable random set. The last expression can then be either directly incorporated into the overall model for $pr(Q|Y)$ or used in its own right for correlation.

At present, models are being established for the fuzzy set membership functions involved. Numerical tables of correlation sensitivities to parameter changes, operator variability and confidence levels are planned.

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AN APPLICATION OF THE CANONICAL MAPPINGS RELATING FUZZY SET AND PROBABILITY THEORY TO NAVAL OCEAN-SURVEILLANCE CORRELATION IN A RULE-BASED CONTEXT

DR. I. R. GOODMAN, CODE 7232
DEVELOPMENT SUPPORT BRANCH
SURVEILLANCE SYSTEMS DEPARTMENT
NAVAL OCEAN SYSTEMS CENTER, BAYSIDE
SAN DIEGO, CALIFORNIA 92152

APRIL, 1981

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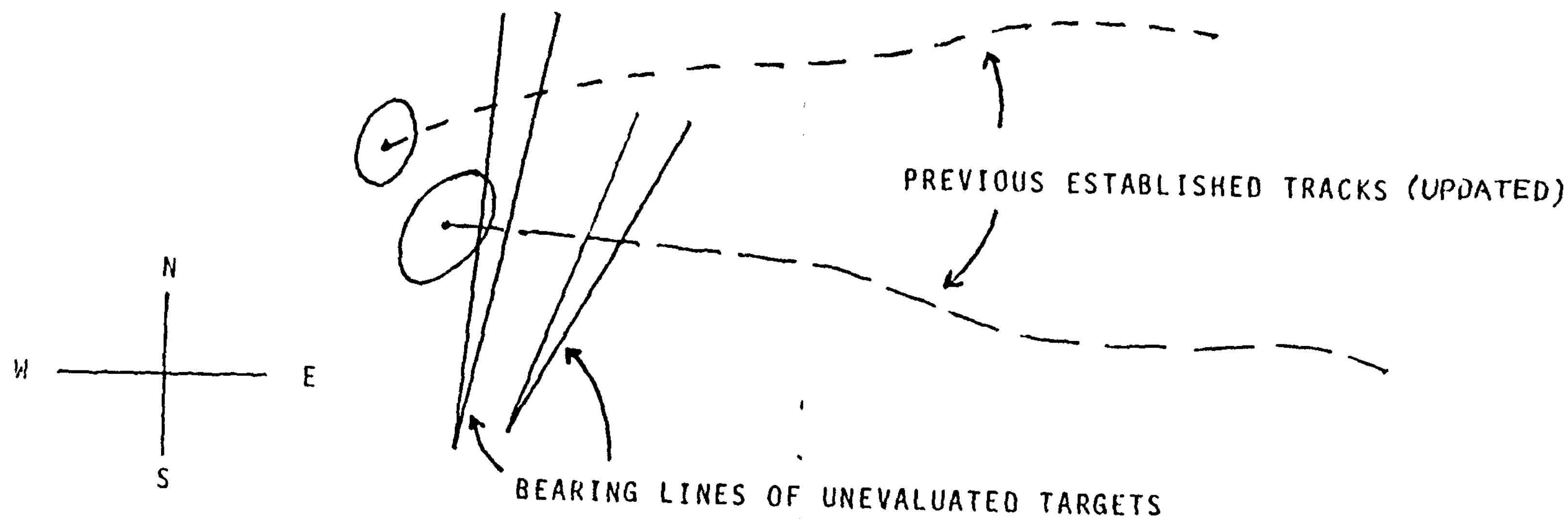
- TYPICAL CORRELATION PROBLEM



INFORMATION AVAILABLE DESCRIBING EVALUATED AND UNEVALUATED TARGETS INCLUDES CLASSIFICATION, BEARINGS, FREQUENCY INFORMATION AND LITERAL COMMENTS SUCH AS SEA STATE LEVELS, MERCHANT SHIPPING DENSITY, ETC.

DEFINITION OF THE SCORES

$$J(\mathring{Q}^{(j)}, \mathring{z}^{(j)}) = -2 \log \mathrm{pr}(Q^{(j)} = \mathring{Q}^{(j)} \mid z^{(j)} = \mathring{z}^{(j)})$$

$$J'(\mathring{Q}^{(j)}, \mathring{z}^{(j)}) = -2 \log \mathrm{pr}(z^{(j)} = \mathring{z}^{(j)} \mid Q^{(j)} = \mathring{Q}^{(j)})$$

TAKING EXPECTATIONS

$$I(\mathring{Q}^{(j)}, \mathring{\mathring{Q}}^{(j)}) = E(J(\mathring{Q}^{(j)}, z^{(j)}) \mid Q^{(j)} = \mathring{\mathring{Q}}^{(j)})$$

$$I'(\mathring{Q}^{(j)}, \mathring{\mathring{Q}}^{(j)}) = E(J'(\mathring{Q}^{(j)}, z^{(j)}) \mid Q^{(j)} = \mathring{\mathring{Q}}^{(j)})$$

$Q^{(j)}$ IS PARTITIONING OF ALL DATA UP TO t_j

$z^{(j)}$ IS ACCUMULATED DATA, INCLUDING $Z^{(j)}$ AND $Y^{(j)}$, UP TO t_j

I, I' ARE CROSS ENTROPY MEASURES

- STRUCTURE OF THE SCORES

J' IS CONSIDERABLY SIMPLER IN FORM THAN J :

$$J'(Q^{(j)}, \underline{g}^{(j)}) = -2 \log pr(Z_{+}^{(j)} \mid Q^{(j)})$$
$$-2 \log pr(Z_0^{(j)} \mid Q^{(j)}) -2 \log pr(Y^{(j)} \mid Q^{(j)})$$

= GEOLOCATION TARGET DATA TERM

+

FALSE ALARM TERM

+

NON-GEOLOCATION TARGET ATTRIBUTE TERM

- GEOLOCATION DATA TERM

$$
\begin{aligned}
&-2 \log \mathrm{pr}(Z_+(j) \mid Q(j)) \\
&= -2 \log \Bigg(\prod_{\substack{1 \le i \\ \text{such that} \\ Z_i(j) = \phi}} \; \prod_{\substack{0 \le a \le j \\ \text{such that} \\ Z_{ia} \ne \phi}} \mathrm{pr}(Z_{ia} \mid Z_i(a-1), Q(j)) \Bigg) \\
&= \sum_{\substack{1 \le i \\ \text{such that} \\ Z_i(j) \ne \phi}} \left(L_i(j) \right) ,
\end{aligned}
$$

$$
L_i(j) = -2 \log \mathrm{pr}(Z_i(j) \mid Q(j)) = \sum_{\substack{0 \le a \le j \\ \text{such that} \\ Z_{ia} \ne \phi}} (L_{ia}) ,
$$

$$
L_{ia} = -2 \log \mathrm{pr}(Z_{ia} \mid Z_i(a-1), Q(j)) .
$$

$(Z_{ia} \mid Z_i^{(a-1)}, Q^{(j)})$ is the r.v. of innovations at t_a for track set i based on $Q^{(j)}$, $j \geq a$, and is distrib. as $N(\mu_{ia}^{(j)}, \Sigma_{ia}^{(j)})$, where S_{ia}

$$\mu_{ia}^{(j)} = E(Z_{ia} \mid Z_i^{(a-1)}, Q^{(j)}) = B_{ia} \cdot \hat{X}_{i,a-1,a}^{(j)},$$

$$\hat{X}_{i,a-1,a}^{(j)} = E(X_{ia} \mid Z_i^{(a-1)}, Q^{(j)})$$

is the optimal estimator of X_{ia} given data $Z_i^{(a-1)}$ and $Q^{(j)}$;

$$\Sigma_{ia}^{(j)} = \mathrm{Cov}(Z_{ia} \mid Z_i^{(a-1)}, Q^{(j)}) = B_{ia} \cdot \Lambda_{i,a-1,a}^{(j)} \cdot B_{ia}^T + R_{ia}$$

is the covariance matrix of error of the optimal estimator of X_{ia} given $Z_i^{(a-1)}$ and $Q^{(j)}$.

$\hat{X}_{i,a-1,a}^{(j)}$ and $\Lambda_{i,a-1,a}^{(j)}$ may be obtained recursively as outputs of the standard Kalman filter for a linear Gauss-Markov data measurement and target motion model, relative to perceived track set i.

$L_i^{(j)}$ is the goodness-of-fit of the geolocation data for track set i up to t_j, under $Q^{(j)}$ (some functional dependencies being omitted for simplicity), while L_{ia} is the corresponding goodness-of-fit at t_a. L_{ia} is seen to decompose as :

$$L_{ia} = L'_{ia} + L''_{ia} \quad ,$$

$$L'_{ia} = s_{ia} \cdot \log 2\pi + \log \det(\Sigma_{ia}^{(j)}) \ ,$$

$$L''_{ia} = \nu_{ia}^{(j)T} \cdot \Sigma_{ia}^{(j)-1} \cdot \nu_{ia}^{(j)} \quad .$$

$$\nu_{ia}^{(j)} = z_{ia} - \mu_{ia}^{(j)}$$

is the innovations for track set i at t_a (based on $Q^{(j)}$).

In computing the matrix inverse and the determinant of $\Sigma_{ia}^{(j)}$, as well as $\mu_{ia}^{(j)}$, for better efficiency, care must be taken in considering the cases $s_{ia} > m$ and $s_{ia} \leq m$, separately.

FALSE ALARM TERM

$$-2 \log \mathrm{pr}(Z_0^{(j)} | Q^{(j)}) = \sum_{0 \le a \le j} \sum_{\substack{1 \le k \le g \\ \text{such that} \\ Z_{0ak} \ne \phi}} L_{0,a,k}$$

where

$$L_{0,a,k} = -2 \log \mathrm{pr}(Z_{0,a,k} | Q^{(j)}) = L'_{0,a,k} + L''_{0,a,k}$$

$$L''_{0,a,k} = \sum_{h=1}^{f_{a,k}} (Z_{0,a,k,h} - \theta_{a,k})^T \cdot M_{a,k}^{-1} \cdot (Z_{0,a,k} - \theta_{a})$$

$$L'_{0,a,k} = f_{a,k} \cdot (r_{a,k} \cdot \log 2\pi + \log \det M_{a,k}).$$

- NON-GEOLOCATION TARGET ATTRIBUTE TERM

$$-2 \log pr(Y^{(j)} \mid Q^{(j)}) = \sum_{\substack{1 \le i \\ \text{such that} \\ Y_i(j)}} \left(K_i^{(j)} \right),$$

$$K_i(j) = -2 \log pr(Y_i(j) \mid Q^{(j)}) = \sum_{\substack{0 \le \alpha \le j \\ \text{such that} \\ Y_{i\alpha} \neq \phi}} \left(K_{i\alpha} \right)$$

$$\begin{aligned} K_{i\alpha} &= -2 \log pr(Y_{i\alpha} \mid Y_i(\alpha-1), Q(j)) \\ &= -2 \log \left(\sum_{\substack{\text{over all} \\ H_i \in C}} \left(pr(Y_{i\alpha} \mid H_i, Y_i(\alpha-1), Q(j)) \cdot pr(H_i) \right) \right) \end{aligned}$$

- CASE 1.

GAUSSIAN APPROXIMATION MADE IN ASSUMPTIONS YIELD A RESULT ANALAGOUS TO — AND SIMPLER THAN — GEOLOCATION TERM IN RECURSIVE FORM (KALMAN FILTER FORMS)

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- PROBABILISTIC FORMULATION OF CORRELATION - DIRECT FORM

$$\Pr\left(Q^{(j)} \mid Y^{(j)}\right)$$

$$= M_j \cdot \Pr\left(Y^{(j)} \mid Q^{(j)}\right)$$

- M_j IS A FUNCTION OF $Y^{(j)}$ AND $Q^{(j)}$.

  $\Pr\left(Y^{(j)} \mid Q^{(j)}\right)$ IS A PRODUCT OF GOODNESS-OF-FIT FACTORS:

  GEOLOCATION

  FALSE ALARMS

  ATTRIBUTES

- PROBABILISTIC FORMULATION OF CORRELATION - RECURSIVE FORM

$$Pr\,(Q^{(j)} \mid Y^{(j)})$$

$$= K_j \cdot L_j \cdot Pr(Q^{(j-1)} \mid Y^{(j-1)})$$

- $Q^{(j)}$ IS DATA PARTITIONING - CORRELATION UP TO t_j ,
  $Y^{(j)}$ IS DATA UP TO t_j
- K_j IS A COMPUTABLE PRODUCT OF GOODNESS-OF-FIT FACTORS: GEOLOCATION (KALMAN FILTER INNOVATION FORMS), FALSE ALARMS (POISSON MODELING, ETC.), <u>ATTRIBUTES</u>.

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- MAIN GOAL:

  TO MODEL PROBABILITIES, LIKELIHOODS OR POSSIBILITIES OF CORRELATIONS BASED ON TIME AVAILABLE OBSERVED ATTRIBUTES AND EXISTING DECISION RULES OBTAINED BY QUERYING EXPERT OPERATORS IN THE FIELD.

NOSC

- BRIEF OUTLINE OF FUZZY SETS AND THEIR RELATION TO PROBABILITY

DEFINITION

- Fuzzy Subset C of ordinary Set X is identified by the membership function

$$\phi_C : X \longrightarrow [0,1]$$

where: for any x in X, $\phi_C(x)$ is a number between 0 & 1 representing the degree of membership of x being in C

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EXAMPLES OF FUZZY SETS

Assuming a poll or sampling survey can be carried out on the meaning of any of the following definitions:

- C is any ordinary Subset of X
- C = All small adult males living in San Diego

  X = All adult males living in California
- C = All ships in a given area which have Type 1 hull profile & are narrow masted

  X = All ships in a given area

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EXAMPLES OF FUZZY SETS (CONTD)

- C = Many

  X = [0,1]

- C = Approximately equal

  X = Two-dimensional Euclidean space

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FUZZY SET MEMBERSHIP CAN BE INTERPRETED IN TERMS OF TRUTH LEVELS

$$
\begin{tikzcd}
\left(\begin{array}{c}\text{BASE}\\ \text{SPACE}\\ X\end{array}\right) \arrow[rr, "\text{FUZZY SET MEMBERSHIP FUNCTION } \phi_A"] \arrow[dr, "\text{FUZZY SET } A"'] & & {[0,1]} \\
& \left(\begin{array}{c}\text{SPACE P OF}\\ \text{LOGICAL COMBINATIONS}\\ \text{OF PROPOSITIONS}\\ \text{OF THE FORM}\\ \text{'a IS IN A'}\end{array}\right) \arrow[ur, "\text{TRUTH FUNCTION } t"'] &
\end{tikzcd}
$$

FOR ALL x IN X: $\phi_A(x) = t(A(x))$ = TRUTH VALUE OF $A(x)$
= TRUTH VALUE THAT $x \epsilon A$

FOR ALL x,y IN X: $MIN(\phi_A(x), \phi_B(y)) = t(A(x) \ \& \ B(y))$

$MAX(\phi_A(x), \phi_B(y)) = t(A(x) \text{ or } B(y))$

$1 - \phi_A(x) = t\,(\text{not } A(x))$

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BRIEF COMPARISON BETWEEN FUZZY & ORDINARY OPERATIONS

FUZZY	ORDINARY	
A, B fuzzy subsets of X	A, B ordinary subsets of X	
$\phi_A(x) = \alpha$: membership level of x in A is α; $0 \leqslant \alpha \leqslant 1$	Either $x \in A$ ($\phi_A(x) = 1$) or $x \notin A$ ($\phi_A(x) = 0$)	
$\phi_{X \dashv A}(x) = 1 - \phi_A(x)$	$X \dashv A$	complement of A
$\phi_{A \cap B}(x) = \mathrm{MIN}\ (\phi_A(x), \phi_B(x))$	$A \cap B$	intersection of A, B
$\phi_{A \cup B}(x) = \mathrm{MAX}\ (\phi_A(x), \phi_B(x))$	$A \cup B$	union of A, B
$\phi_{A \Rightarrow B}(x) = \phi_{(X \dashv A) \cup B}(x)$	$A \Rightarrow B$	implication of A to B
$\phi_A(x) \leqslant \phi_B(x)$, all x: $A \subseteq B$	$A \subseteq B$	A is subset of B
$\phi_{f(A)}(y) = \sup\limits_{x \in f^{-1}(y)} (\phi_A(x))$	$f(A)$	function of A — pointwise image
$\phi_{A \times B}(x,y) = \mathrm{MIN}\ (\phi_A(x), \phi_B(y))$	$A \times B$	cartesian product of A, B
$\phi_{PROJ_1(A)}(x) = \sup\limits_{(\text{all } y)} (\phi_A(x,y))$; all $(x, y) \in X \times Y$; A is a fuzzy subset of $X \times Y$	$PROJ_1(A)$	x-coordinate projection of A; A is subset of $X \times Y$

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Background

- What are random sets?
  - Any variable set taking different values with various probabilities
  - Random variables & random vectors are special cases of random sets
- Example 1 — A random subset of R^+

  $(3U + 8, 5U + 16V) \cup \{x \mid x \in f^{-1}(U)\}$,
  where U, V are given random variables on R^+

  f is any given function on R^+ to R^+

- Example 2

$[\bar{x} - a, \bar{x} + a]$ is an α-level confidence random interval for μ:

$$\alpha = \Pr(\mu \epsilon [\bar{x} - a, \bar{x} + a] \mid \mu)$$
$$= \Pr([\bar{x} - a, \bar{x} + a] \epsilon C_\mu \mid \mu); \text{ for all } \mu.$$

$\bar{x}$ is sample mean from statistically independent $x_1, \ldots, x_n$.
Each x_i is distributed $N(\mu, \sigma_i^2)$.

μ is unknown & each σ_i^2 is known.

$$\alpha = 2\Psi\left(a\Big/\left((1/n)\sum_{i=1}^{n}\sigma_i^2\right)^{1/2}\right), \quad \Psi(x) = \left(1/\sqrt{2\pi}\right)\int_0^x e^{-(1/2)t^2}\,dt.$$

a is a known constant value.

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- Examples Which Have Been Solved in Detail

  - V_1 is a small positive real number such that V_2, another positive number, is approximately equal to V_1. V_3, another positive real number, is much larger than both. Determine size of V_3.

  - Most ships spotted in a surveillance area of interest appeared to have tall masts. It is also known from previous experience that most of the tall masted ships spotted there are enemy ships. How many ships in the area are enemy ones & how many are tall masted enemy ships?

  - Five individuals are polled concerning the age of Kati Xequoz. A: "It is not quite true she is very old." B: "It is not true she is young." C: "I believe she is most likely between 40 & 45 years of age." D: "Based on dental records, the probability that she is between 44 & 47 years is at least 0.9." Based on the nuances of the statements, how old is Kati?

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- Another Example
  - Premise
    - From experience, a sensor operator knows when monitoring a target if attribute A_1 (maneuvering) is strongly present, then probably attribute A_2 (enemy awareness of being followed) is mildly present
    - A_1 depends on χ^2 — the observed statistical goodness of model fit
    - A_2 depends on q — an m-dimensional vector representing intelligence information concerning the target
    - On one particular occasion, the operator observes A_1 to be moderately present
  - Conclusion

    What can be said about the presence of A_2 during the latter occasion?

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BASIC THEORETICAL RESULTS CONNECTING FUZZY SETS WITH RANDOM SETS

- Well Known
  - Given any random subset Z of space X, there is a unique fuzzy subset A(Z) of X such that

    $A(Z) \approx Z$ i.e.,

    $\phi_{A(Z)}(x) = \Pr(x \in Z)$; for all $x \in X$

- New Results (Goodman, 1976, 1980; Höhle, 1980)
  - Given any fuzzy subset A of space X, there are many random subsets S(A) of X such that

    $A \approx S(A)$, i.e.,

    $\phi_A(x) = \Pr(x \in S(A))$; for all $x \in X$

- New Results (CONTD)
  - In particular, let $S = S_U$, where

    $S_U(A) = \phi_A^{-1}([U, 1])$;

    U is a uniformly distributed R.V. over [0, 1].

  Thus, $A \approx S_U(A)$:

    $\phi_A(x) = Pr(x \in S_U(A))$; all $x \in X$.

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- New Results (CONTD)
  - Or, let $S = T_{(\,)}$, where $T_{(\,)}$ is best illustrated as follows:

    Let $X = \{x_1, x_2, \ldots, x_n\}$ be finite

    Let $\phi_A : X \to [0, 1]$ be arbitrary

    Thus, $\phi_A(x_1), \ldots, \phi_A(x_n)$ are numbers between 0 and 1.

    Then define random subset T_A of X:

    for any (ordinary) subset B of X,

$$Pr\,(T_A = B) = \prod_{x_i \in B} \phi_A(x_i) \cdot \prod_{x_i \in X - B} (1 - \phi_A(x_i))$$

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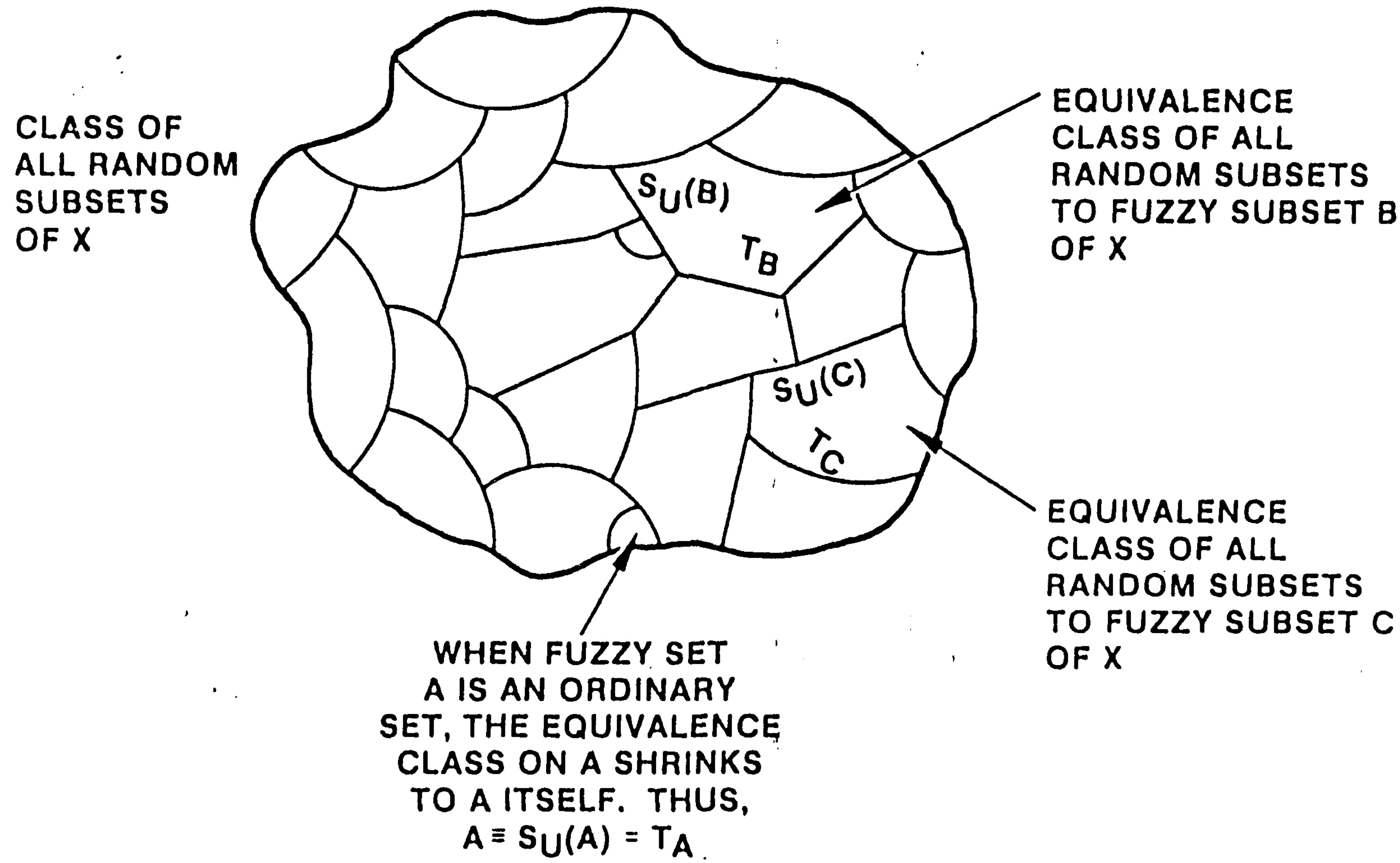
NEW RESULTS (CONTD)

SCHEMATICALLY, THE FOLLOWING PARTITIONING OF THE CLASS OF ALL RANDOM SUBSETS OF X HOLDS:



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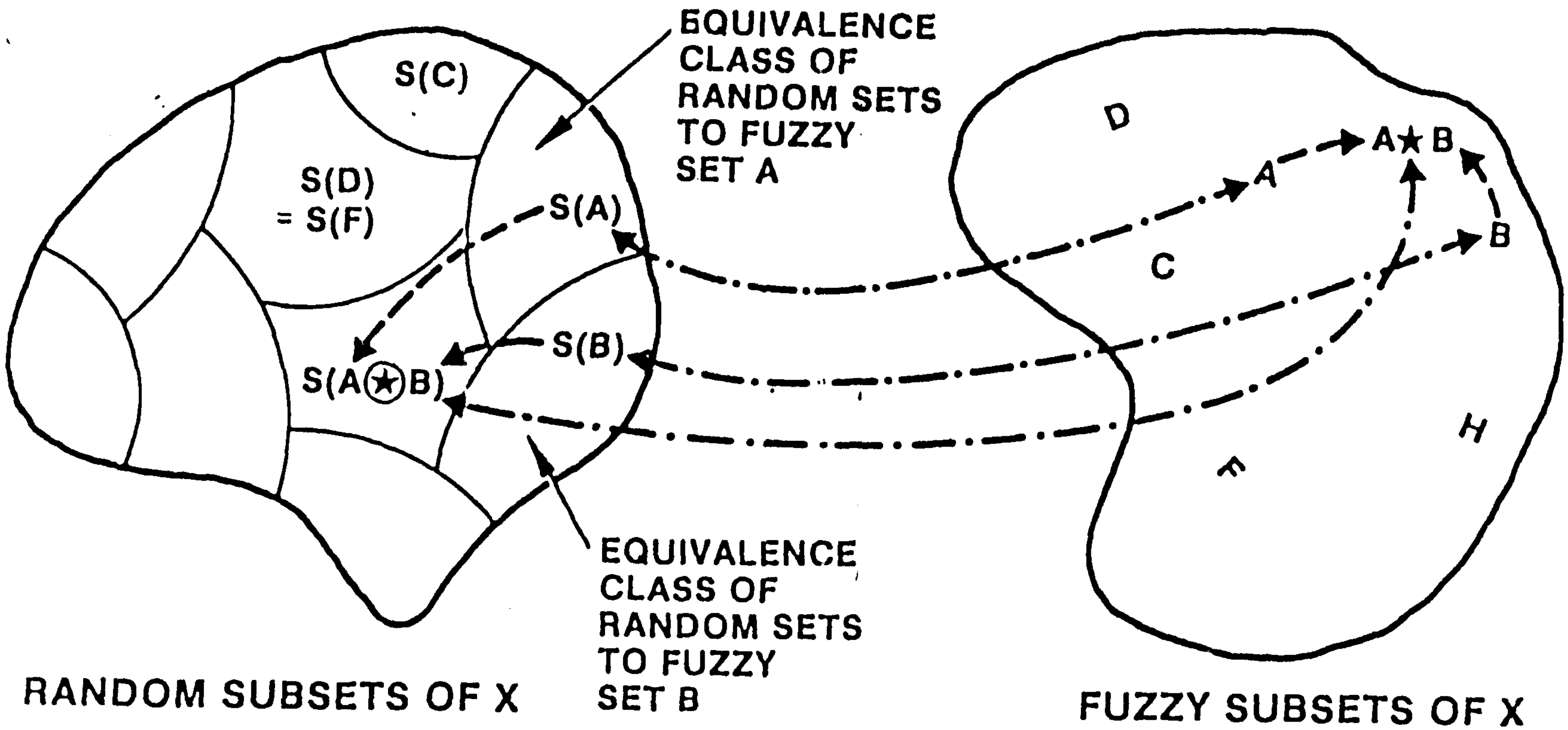
WORK IS PRESENTLY CONTINUING ON DEVELOPING A CHARACTERIZATION OF HOMOMORPHISMS FROM FUZZY SET OPERATIONS TO RANDOM SET ONES



★ IS A BINARY OPERATION BETWEEN FUZZY SETS

⊛ IS A BINARY OPERATION BETWEEN RANDOM SETS

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- New Results (CONTD)
  - S_U is a homomorphism (or nearly so) with respect to all of the basic fuzzy set operations & the corresponding ordinary set ones:
    - $S_U(A \cup B) = S_U(A) \cup S_U(B)$
    - $S_U(A \cap B) = S_U(A) \cap S_U(B)$
    - $S_U(X \dashv A) = X \dashv S_{1-U}(A)$
    - $S_U(A \Rightarrow B) = (S_{1-U}(A) \Rightarrow S_U(B))$
    - $S_U(A \times B) = S_U(A) \times S_U(B)$
    - $A \subseteq B$ IFF ($S_U(A) \subseteq S_U(B)$, all outcomes of U)
    - $S_U(f(A)) = f(S_U(A))$; $S_U(f^{-1}(B)) = f^{-1}(S_U(B))$
    - $S_U(PROJ_1(A)) \subseteq PROJ_1(S_U(A))$

- $T_{(\,)}$ is also a homomorphism with respect to the basic fuzzy set operations:

  - $T_{A \oplus B} = T_A{}' \cup T_B{}''$
  - $T_{A \bullet B} = T_A{}' \cap T_B{}''$
  - $T_{X \dashv A} = 1 - T_A$
  - $T_{A \textcircled{\Rightarrow} B} = T_A{}' \Rightarrow T_B{}''$
  - $T_{A \otimes B} = T_A{}' \times T_B{}''$
  - $T_{A \textcircled{\lambda} B} = (T_\lambda{}' \cap T_A{}'') \cup ((X \dashv T_\lambda{}') \cap T_B{}''')$
  - $T_{f(A)} = f(T_A);\ T_{f^{-1}(B)} = f^{-1}(T_B)$
  - $Pr\,(x \in T_{PROJ_1(A)}) \leqslant Pr\,(x \in PROJ_1\,(T_A));$ all $x \in X$

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APPLICATIONS OF THE RESULTS TO FUZZY LOGIC

- Premise: $V \in R^m$ representing m various unknown quantities is restricted by:

  (V has property A_1 to at least degree α_1)

  & ....

  & (V has property A_n to at least degree α_n)

  & (V is covered by random set Z_1 with at least probability β_1)

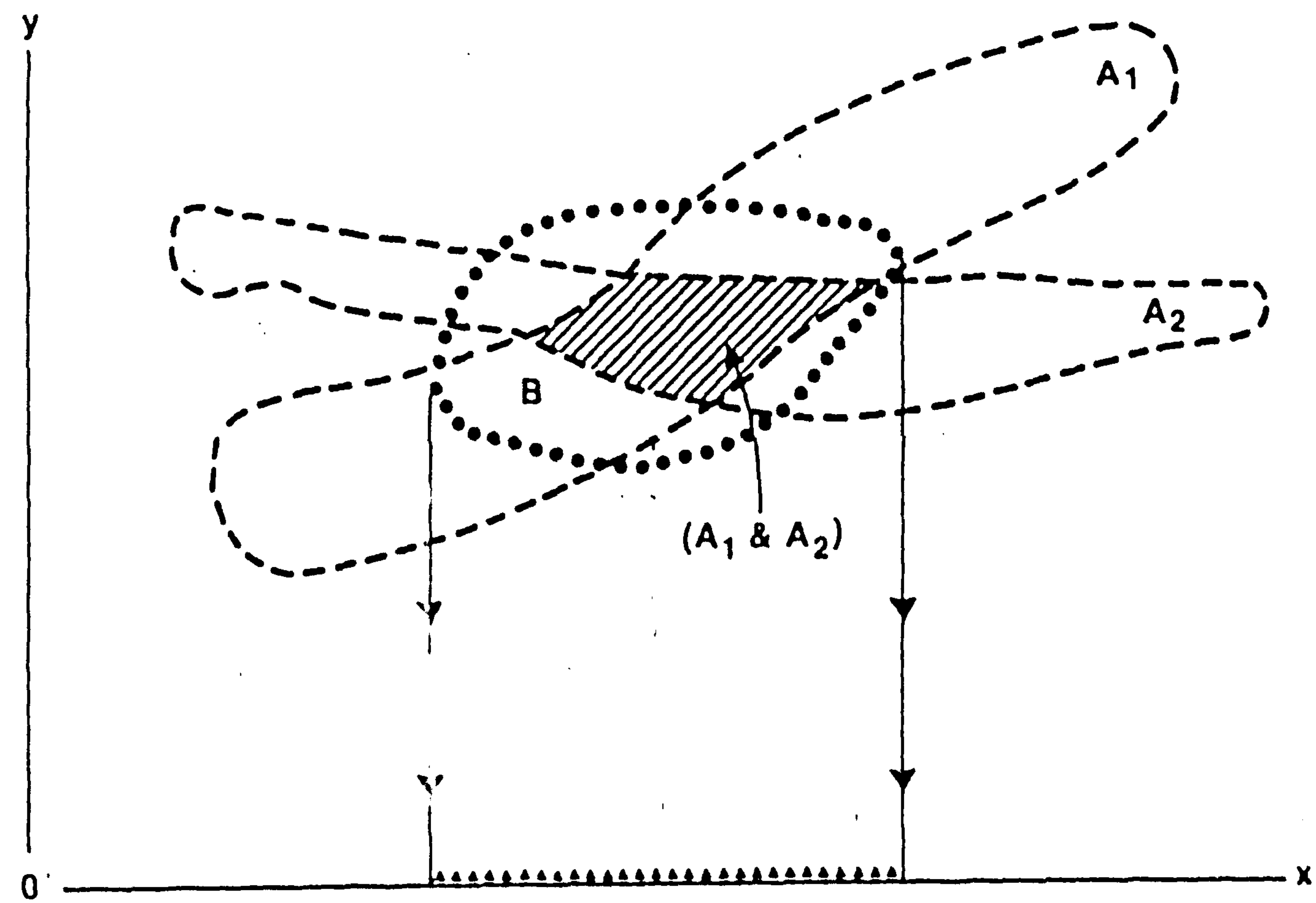
  & ....

  & (V is covered by random set Z_r with at least probability β_r)

- Conclusion 1: Determine a computationally feasible restriction on V.
- Conclusion 2: Determine as above for any specified components of V.

NOSC

GEOMETRIC INTERPRETATION OF CONJUNCTIVE PREMISES



Conclusion B, with boundary ••••, contains as a proper subset, premise (A_1 & A_2), indicated by ////. Projection C = $PROJ_1$ (B) is indicated by ▴▴▴▴▴.

- Translation of premise into fuzzy set & random set notation

$$(\phi_{A_1}(V) \geqslant \alpha_1) \ \& \ldots \& \ (\phi_{A_n}(V) \geqslant \alpha_m) \ \& \ (\Pr(V \in Z_1) \geqslant \beta_1)$$
$$\& \ldots \& \ (\Pr(V \in Z_r) \geqslant \beta_r)$$

- Conclusion 1:

Independent of any joint distribution assigned to the U_i from the relations $A_i \approx S_{U_i}(A_i)$ (new result) and/or to the Z_j:

$$\alpha \leqslant \phi_B(V) = \Pr(V \in S_U(B)),$$

where
V is arbitrary satisfying the premise &

U is uniform R.V. over [0, 1]

$\alpha = \mathrm{MIN}(\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_r)$,

$B = A_1 \cap \ldots \cap A_n \cap A(Z_1) \cap \ldots \cap A(Z_r)$

- Conclusion 2

  Similarly, independent of all joint distributions assigned to the U_l and/or to the Z_j,

  Conclusion:

  $$\alpha \leqslant \phi_C(V_1) = Pr\,(V_1 \epsilon\, S_U(C)) \leqslant Pr\,(V_1 \epsilon\, PROJ_1\,(S_U(B)))$$

  where

  V_1 is the first component of V and arbitrary satisfies the premise

  and where

  $$C = PROJ_1(B)$$

- Remarks
  - $\phi_B(V) = MIN\,(\phi_{A_1}(V), \ldots, \phi_{A_n}(V), Pr\,(V \epsilon\, Z_1), \ldots, Pr\,(V \epsilon\, Z_r))$
  - $S_U(B) = S_U(A_1) \cap \ldots \cap S_U(A_n) \cap S_U(A(Z_1)) \cap \ldots \cap S_U(A(Z_r))$
  - The conclusion can be shown to be a uniformly most accurate one and can be modified for arbitrary combinations of '&' and 'or' in the premise

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- Additional Remarks
  - If the U_i and the Z_j can all be chosen statistically independent (there may be some problem in doing this) then the following conclusion also holds for V:

$$\gamma \leqslant \Pr\left(V \in \bigcap_{i=1}^{n} S_{U_i}(A_i) \cap \bigcap_{j=1}^{r} Z_j\right),$$

where

$$\gamma = \prod_{i=1}^{n} \alpha_i \cdot \prod_{j=1}^{r} \beta_j$$

- In any case, B can be considered an α-level fuzzy confidence set for V, & simultaneously, $S_U(B)$ can be considered an α-level random confidence set for V.
- In all of the above results, S_U can be replaced by $T_{(\,)}$ with corresponding changes

SUMMARY OF APPLICATIONS OF RESULTS TO LOGIC

- Given a premise of possibly mixed propositional & probabilistic statements restricting an otherwise unknown vector of numerical quantities, equivalent fuzzy & random confidence sets for this vector can be constructed
- Similarly, equivalent fuzzy & random confidence sets may be constructed for any of the components of the unknown vector
- The operations involved in the constructions are mainly intersections, minima, or suprema

NOSC

COMPLETE SOLUTION OF PREVIOUS EXAMPLE

- Premise
  - From experience, a sensor operator knows when monitoring a target if attribute A_1 (maneuvering) is strongly present, then probably attribute A_2 (enemy awareness of being followed) is mildly present
  - A_1 depends on χ^2 — the observed statistical goodness of model fit
  - A_2 depends on q — an m-dimensional vector representing intelligence information concerning the target
  - On one particular occasion, the operator observes A_1 to be moderately present
- Conclusion

  What can be said about the presence of A_2 during the latter occasion?

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- Define or assume

  - $V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$, where $V_1 = \chi^2 \in R^+$ and $V_2 = q \in R^m$
  - $\phi_{A_1} : R^+ \longrightarrow [0, 1]$ is onto
  - $A_3 = A_1$ very strongly present: $\phi_{A_3}(V_1) = \phi_{A_1}^{\,3}(V_1)$
  - $A_4 = A_2$ mildly present: $\phi_{A_4}(V_2) = \phi_{A_2}^{\,1/2}(V_2)$
  - A_5 = probably: $\phi_{A_5}(x) = \mathrm{MAX}(\beta \cdot x, 1 - \beta)$;

    β fixed at 0.5, say; $x \in [0, 1]$
  - A_6 = probably A_2 mildly present: $\phi_{A_6}(V_2) = \phi_{A_5}(\phi_{A_4}(V_2))$
  - A_7 = if ( ) then ( ): $\phi_{A_7}(x,y) = \mathrm{MAX}(1 - x, y)$; $x,y \in [0, 1]$
  - $A_8 = A_3 \Rightarrow A_6$: $\phi_{A_8}(V) = \phi_{A_7}(\phi_{A_3}(V_1), \phi_{A_6}(V_2))$

- Conclusion
  - $\alpha \leqslant \phi_C(V_2) = Pr\ (V_2 \epsilon\ S_U(C)) \leqslant Pr\ (V_2 \epsilon\ PROJ_2(S_U(B)))$,

    $\alpha = MIN\ (\alpha_1, \alpha_2)$,

    $C = PROJ_2\ (B)$,

    $\phi_C(V_2) = \underset{V_1 \epsilon R^+}{SUP}\ (\phi_B(V)) = MAX\ (x_0, \phi_{A_6}(V_2))$,

    $\phi_{A_6}(V_2) = MAX\ (\beta \cdot \phi_{A_2}{}^{1/2}(V_2), 1 - \beta)$,
  - α-level fuzzy confidence set for V_2 is:

$$\{V_2 \mid \alpha \leqslant \phi_C(V_2), V_2 \epsilon R^m\} = \begin{cases} R^m, \text{ i.e., no confidence, if } 0 \leqslant \alpha \leqslant \gamma(\beta) \\ \{V_2 \mid (\alpha/\beta)^2 \leqslant \phi_{A_2}(V_2), V_2 \epsilon R^m\}, \text{ if } \gamma(\beta) < \alpha \leqslant 1, \end{cases}$$

$\gamma(\beta) = MAX\ (1 - \beta, x_0)$

- α-level random confidence set for V_2 is the same & is describable in terms of $S_U(C)$:

$$\{V_2 \mid \alpha \leqslant \Pr(V_2 \in S_U(C)),\ V_2 \in R^m\}$$

- Random set $S_U(C)$ is explicitly

$$S_U(C) = \phi_C^{-1}([U, 1]) = \begin{cases} R^m, & \text{if } 0 \leqslant U \leqslant \gamma(\beta) \\ \phi_{A_2}^{-1}([(U/\beta)^2, 1]), & \text{if } \gamma(\beta) < U \leqslant 1 \end{cases}$$

- Linguistically, C may be interpreted as

  $C = A_2$ is somewhat less than probably mildly present
  = somewhat less than A_6,

  where 'somewhat less than' is measured inversely to the degree that MAX $(x_0, \phi_{A_6}(V_2))$ exceeds $\phi_{A_6}(V_2)$

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- Translation of premise

$$(\phi_{A_8}(V) \geqslant \alpha_1) \,\&\, (\phi_{A_1}(V_1) \geqslant \alpha_2)$$

- In deriving conclusion, fuzzy set B is computed

$B = A_8 \cap A_1,$

$$\phi_B(V) = \begin{cases} \phi_{A_1}(V_1) & \text{IFF } 0 \leqslant \phi_{A_1}(V_1) \leqslant \text{MAX}\,(x_0, \phi_{A_6}(V_2)) \\ 1 - \phi_{A_1}^3(V_1) & \text{IFF } x_0 \leqslant \phi_{A_1}(V_1) \leqslant (1 - \phi_{A_6}(V_2))^{1/3} \\ \phi_{A_6}(V_2) & \text{IFF MAX}\,((1 - \phi_{A_6}(V_2))^{1/3}, \phi_{A_6}(V_2)) \\ & \qquad \leqslant \phi_{A_1}(V_1) \leqslant 1 \end{cases}$$

where

x_0 is the solution of the equation

$$x_0 = 1 - x_0^3$$

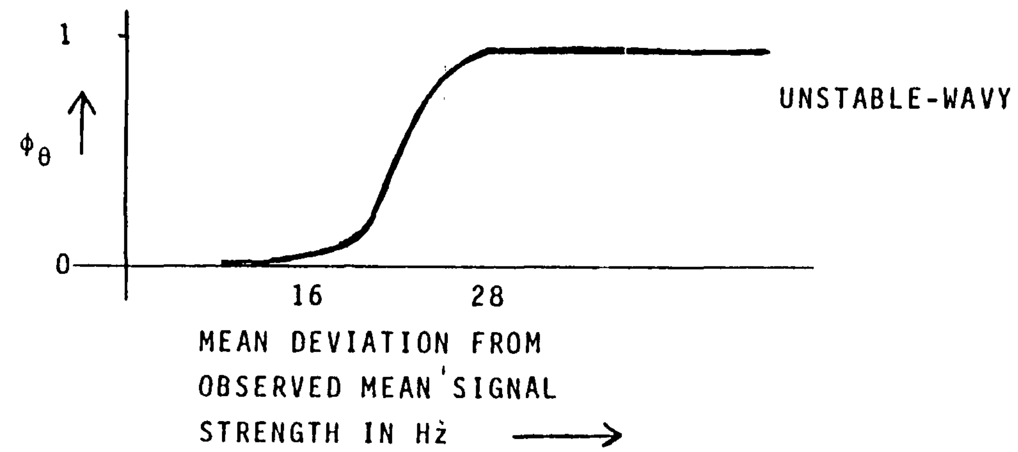
- MATHEMATICALLY, EACH DETERMINED VALUE OF AN ATTRIBUTE OF A GIVEN TARGET, IS CONSIDERED AS A FUZZY SET.

- THESE FUZZY SETS WILL HAVE MEMBERSHIP FUNCTIONS DETERMINED BY QUERYING EXPERIENCED PERSONNEL.

- EXAMPLE OF TERMS

ATTRIBUTE OF TARGET A	TYPICAL VALUE, LITERAL OR NUMERICAL θ	TYPICAL CONFIDENCE α
BEARING LINE	158^{0}	$\pm 10^{0}$, for 95%
CLASSIFICATION	TIBETAN TYPE 4, BUT COULD BE TYPE 3, OR, LESS LIKELY, TYPE 5	MEDIUM
RANGE LIMITATION	1000 MILES	95% LEVEL
FUNDAMENTAL STRENGTH	198 Hz	±10Hz for 90%
SIGNAL STRENGTH	MEDIUM	HIGH
SIGNAL STABILITY	UNSTABLE-WAVY	HIGH
OBSERVED MANEUVERING	SOME	MEDIUM
OBSERVED HARMONICS	117, 234	LOW

- EXAMPLE 1


1
ϕ_θ
0
16
28
MEAN DEVIATION FROM
OBSERVED MEAN SIGNAL
STRENGTH IN Hz
UNSTABLE-WAVY

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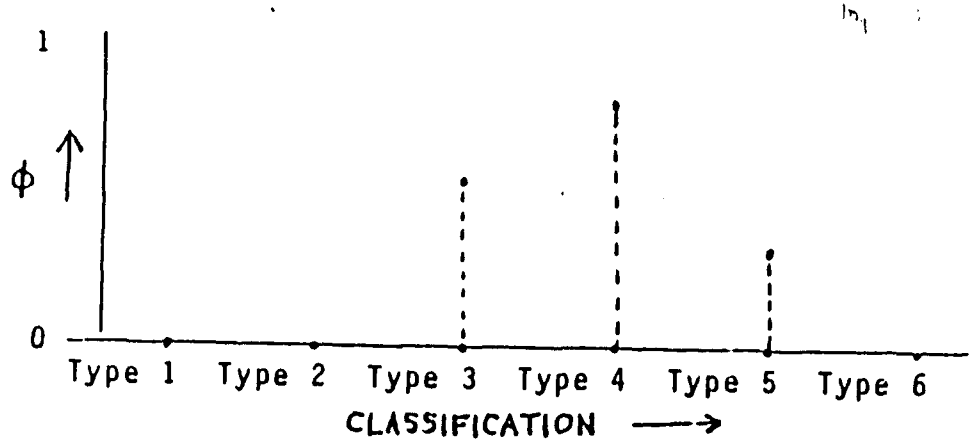
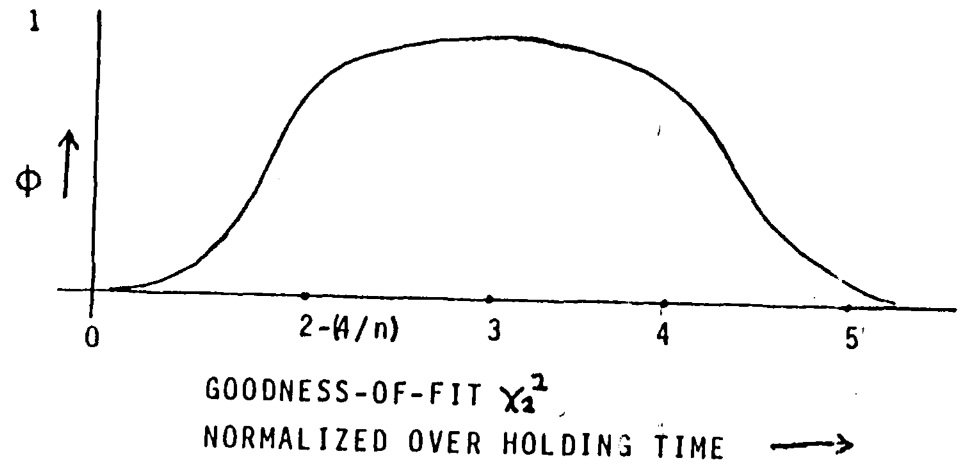
- EXAMPLE 2



A SHIP TYPE 4
BUT COULD BE
TYPE 3 OR LESS
LIKELY, TYPE 5.

- EXAMPLE 3



SOME MANEUVERING
(FOR STRAIGHT LINE
2 DIMENSIONAL
MOTION)

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- OBSERVED DATA STRUCTURED IN FUZZY SET FORM

FOR EACH $i \epsilon W_3, j \epsilon W_2$, $\exists\ K_{ij}$, $\phi \subseteq K_{ij} \subseteq K$,

SUCH THAT:

$$\underset{K \epsilon K_{ij}}{\&} \Big(\text{CONFID} \big[\text{OBJECT } i \text{ POSSESSES ATTRIBUTE } A_k \text{ AT LITERAL VALUE } \theta_{k,i,j} \in M_k \big] \Big) \geq \alpha_{k,i,j} \quad ;$$

$$1 \geq \alpha_{k,i,j} \geq 0$$

NOSC

- EQUIVALENTLY,

$$\underset{\substack{k \in K_{ij}, \\ i \in W_3, j \in W_2}}{\&} \left(\phi_{B_{\theta_{k,i,j}}}(V_k) \geq \alpha_{k,i,j} \right) ,$$

WHERE $B_{\theta_{k,i,j}}$ IS THAT FUZZY SUBSET OF N_k,

WITH MEMBERSHIP FUNCTION $\phi_{B_{\theta_{k,i,j}}} : N_k \rightarrow [0,1]$,

CORRESPONDING TO $\theta_{k,i,j}$.

- EXAMPLES OF TYPICAL RULES

- IF TWO NEW TARGETS OF INTEREST HAVE BEARING LINES WHICH DO NOT INTERSECT IN THE FEASIBLE REGION (WELL-DEFINED), THEN THE TARGETS CANNOT CORRELATE.

- IF TWO NEW TARGETS MATCH REASONABLE WELL ON FUNDAMENTAL SIGNALS, HAVE A FEASIBLE BEARING LINE INTERSECTION, BOTH HAVE GOOD SIGNAL STRENGTHS, AND ALSO MATCH ON TWO HARMONICS, THEN WE CAN SAFELY SAY WITH HIGH CONFIDENCE THAT THEY REPRESENT THE SAME PLATFORM.

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- EXPERT-QUERIED MODUS PONENS MATCHING RULES IN FUZZY SET FORM

FOR EACH $\ell \in W_1$, $\exists\ W_{3,\ell} \subseteq W_s$,

INDEX SETS $M'_{\ell,i}$, FOR $i \in W_{3,\ell}$; $M''_{\ell,b,i}$, FOR

$b \in M'_{\ell,i}$; AND $J_\ell \subseteq J$ SUCH THAT :

$$\text{CONFID}\left\{\underline{\text{IF}}\ \left\{ \underset{\substack{b \in M'_{\ell,i},\\ i \in W_{3,\ell}}}{\&} \left(\underset{a \in M''_{\ell,b,i}}{\text{Or}} \right.\right.\right.$$

$\Big[$ OBJECT i POSSESSES ATTRIBUTE $A_{\gamma_1(\ell,a,b)}$

$$\left.\left.\left.\text{AT LITERAL VALUE } \theta_{\gamma_1(\ell,a,b)} \in M_{\gamma_1(\ell,a,b)}\right]\right)\right\}$$

$\underline{\text{THEN}}$ $\Big\{$ PARTITIONING Q IS DELINEATED

BY POSSIBILITY DISTRIBUTION D_ℓ OVER

$$J_\ell \Big\}\Big\} \geq \beta_\ell$$

NOSC

- EQUIVALENTLY,

$$\mathop{\&}_{\ell \in W_1} \left[\left\{ \phi_{\Rightarrow} \left(\phi_{\substack{\&\\ \left(b \in M'_{\ell,i}\right) \\ \left(i \in W_{3,\ell}\right)}} \left(\phi_{\substack{OR \\ \left(a \in M''_{\ell,b,i}\right)}} \left(\phi_{B_{\theta_{\gamma_2(\ell,a,b)}}} \left(V_{\gamma_1(\ell,a,b)} \right)\right)\right), \right.\right.\right.$$

$$\left.\left.\left. \phi_{D_\ell}(Q) \right) \right\} \geq \beta_\ell \right]$$

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- WE APPLY THE RESULTS OF CONCLUSIONS 1 AND 2 TO THE SITUATION HERE WHERE THE PREMISE CONSISTS OF ALL THE RULES AND OBSERVED DATA CONJOINED IN FUZZY SET FORMS WITH APPROPRIATE CONFIDENCES.

- IN THIS CASE, THE VARIABLE V (OR UNKNOWN PARAMETER VECTOR) CONSISTS OF ALL THE MEASURABLE QUANTITIES THAT GAVE RISE TO THE DATA AND THE UNMEASURABLE TRUE DATA PARTITIONING OR CORRELATION Q:

$$\phi_B(V) \geq \tau$$

- CONCLUSION 2 YIELDS, LETTING $V_1 \equiv Q$, THE FUZZY SET **C**, A PROJECTION OF B, WHICH DELINEATES Q.

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- THE FINAL FORM IS

$$\phi_C(Q) \geq \tau \quad ,$$

WHERE CONFIDENCE τ IS ALSO DETERMINED

- THE ABOVE EXPRESSION, USING PREVIOUS CANONICAL MAPPING, IS EQUIVALENT TO THE PROBABILITY STATEMENT

$$Pr(Q \in S_U(C) \mid Y) \geq \tau$$

(OMITTING TIME INDICES)

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- SUMMARY

- A PROCEDURE IS OUTLINED WHICH DETERMINES THE MOST POSSIBLE DATA ASSOCIATIONS (CORRELATIONS) GIVEN THE OBSERVED DATA.

- THE RES[illegible] IS COMPLETELY ANALAGOUS TO CONCLUSIONS 1 AND 2 - APPLICATIONS OF THE PREVIOUSLY DISCOVERED CANONICAL MAPPING BETWEEN FUZZY AND RANDOM SETS TO FUZZY LOGIC.

- THE PREMISE OF THE ARGUMENT CONSISTS OF A CONJUNCTION OF MODUS PONENS RULE BASED - EXPERT-QUERIED STATEMENTS IN FUZZY SET FORM AND OBSERVED DATA.

- IN CARRYING OUT THE COMPUTATIONS, THE RELEVANT FUZZY SETS MUST BE PROPERLY MODELED, ALL CONFIDENCES OBTAINED, AND ALL MIN-MAX SEARCH OPERATIONS COMPLETED.

- FUTURE WORK WILL BE DIRECTED TOWARD EXAMPLES AND DEVELOPMENT OF SENSITIVITIES TO VARYING INPUT PARAMETERS AND OPERATORS' JUDGMENTS.